8.1 (b) $e^{-10t}u(t)$

$u(t)=0$, for $t<0$
$u(t)=1$, for $t>0$

$$X(s)=\int_{-\infty}^{\infty} e^{-10t}u(t)\,e^{-st}\,dt$$
$$=\int_{0}^{\infty} e^{-10t}e^{-st}\,dt$$
$$=\int_{0}^{\infty} e^{-(s+10)t}\,dt$$
$$=-\frac{1}{s+10}e^{-(s+10)t}\Big|_{0}^{\infty}$$
$$=-\frac{1}{s+10}\left(\lim_{t\to\infty}e^{-(s+10)t}-e^{-(s+10)\cdot 0}\right)$$
$$=-\frac{1}{s+10}(0-1)$$
$$=\frac{1}{s+10}$$

(c) $e^{-10t}\cos(3t)u(t)$

$\cos(3t)=(e^{j3t}+e^{-j3t})/2$ by Euler identity (you can prove it by Taylor expansion)

$$X(s)=\int_{-\infty}^{\infty} e^{-10t}\cos(3t)u(t)\,e^{-st}\,dt$$
$$=\int_{0}^{\infty} e^{-10t}\cos(3t)u(t)\,e^{-st}\,dt$$
$$=\int_{0}^{\infty}\frac{e^{-10t}(e^{j3t}+e^{-j3t})}{2}e^{-st}\,dt$$
$$=\frac{1}{2}\int_{0}^{\infty}e^{-(s+10-3j)t}\,dt+\frac{1}{2}\int_{0}^{\infty}e^{-(s+10+3j)t}\,dt$$

$$= \frac{1}{2}\frac{1}{-(s+10-3j)}\left(\lim_{t\to\infty} e^{-(s+10-3j)t} - e^0\right) + \frac{1}{2}\frac{1}{-(s+10+3j)}\left(\lim_{t\to\infty} e^{-(s+10+3j)t} - e^0\right)$$

$$= \frac{1}{2(s+10-3j)} + \frac{1}{2(s+10+3j)}$$

$$= \frac{s+10+3j+s+10-3j}{2(s+10-3j)(s+10+3j)}$$

$$= \frac{2(s+10)}{2((s+10)^2+3^2)}$$

$$= \frac{s+10}{(s+10)^2+3^2}$$

(d) $e^{-10t}\cos(3t-1)u(t)$

$\cos(3t-1) = \cos(3t)\cos 1 + \sin(3t)\sin 1$ by trignometric Property

$$X(s) = \int_{-\infty}^{\infty} e^{-10t}\cos(3t-1)u(t)e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} e^{-10t}(\cos(3t)\cos 1 + \sin(3t)\sin 1)e^{-st}\,dt$$

$$= \left(\int_0^{\infty}\cos(3t)e^{-(10+s)t}dt\right)\cos 1 + \left(\int_0^{\infty}\sin(3t)e^{-(10+s)t}dt\right)\sin 1$$

$$= \frac{(10+s)\cos 1}{(s+10)^2+9} + \frac{3\cdot\sin 1}{(s+10)^2+9}$$

$$= \frac{(10+s)\cos 1 + 3\sin 1}{(s+10)^2+9}$$ since $\cos wt\, u(t) \leftrightarrow \frac{s}{s^2+w^2}$

$\sin wt\, u(t) \leftrightarrow \frac{w}{s^2+w^2}$

8.4

(a) $x(t) = \left(e^{-bt}\cos^2 wt\right)u(t)$

Table 8.2, $\cos^2 wt \; u(t) \leftrightarrow V(s) = \dfrac{s^2+2w^2}{s(s^2+4w^2)}$

Table 8.1, $\int_{-\infty}^{\infty} e^{-bt} v(t) e^{-st} dt = V(s+b)$

$\Rightarrow \left(e^{-bt}\cos^2 wt\right)u(t) \leftrightarrow V(s+b) = \dfrac{(s+b)^2+2w^2}{(s+b)\left((s+b)^2+4w^2\right)}$

so $X(s) = V(s+b)$

(b) $x(t) = \left(e^{-bt}\sin^2 wt\right)u(t)$

Table 8.2, $v(t) = \sin^2 wt \; u(t) \leftrightarrow V(s) = \dfrac{2w^2}{s(s^2+4w^2)}$

Table 8.1, by $e^{at}x(t) \leftrightarrow X(s-a)$

$\Rightarrow \left(e^{-bt}\sin^2 wt\right)u(t) \leftrightarrow V(s+b) = \dfrac{2w^2}{(s+b)\left[(s+b)^2+4w^2\right]}$

8.10. (b) $X(s) = \dfrac{s+1}{s^3+5s^2+7s}$ Type: two complex poles

$$X(s) = \frac{(s+1)}{s(s^2+5s+7)}$$

$$= \frac{(s+1)}{s\left((s+5/2)^2+7-\frac{25}{4}\right)} = \frac{(s+1)}{s\left((s+\frac{5}{2})^2+\frac{3}{4}\right)}$$

$$= \frac{(s+1)}{s\left(s+\frac{5}{2}+\frac{\sqrt{3}}{2}j\right)\left(s+\frac{5}{2}-\frac{\sqrt{3}}{2}j\right)}$$

$$= \frac{C_1}{s+\frac{5}{2}-\frac{\sqrt{3}}{2}j} + \frac{\bar{C}_1}{s+\frac{5}{2}+\frac{\sqrt{3}}{2}j} + \frac{C_2}{s}$$

for C_1, the pole $p_1 = -\frac{5}{2}+\frac{\sqrt{3}}{2}j$

for C_2, the pole $p_2 = -\frac{5}{2}-\frac{\sqrt{3}}{2}j = \bar{p}_1$

for C_2, the pole $p_3 = 0$

$p_1 = \sigma + jw$ so that $\sigma = -\frac{5}{2}, w = \frac{\sqrt{3}}{2}$

$$C_2 = (s \cdot X(s))\Big|_{s=0} = \frac{0+1}{0^2+5\cdot 0+7} = \frac{1}{7}$$

$$C_1 = \left[(s+\frac{5}{2}-\frac{\sqrt{3}}{2}j)X(s)\right]\Big|_{s=-\frac{5}{2}+\frac{\sqrt{3}}{2}j}$$

$$= \frac{(-\frac{5}{2}+\frac{\sqrt{3}}{2}j+1)}{(-\frac{5}{2}+\frac{\sqrt{3}}{2}j)(-\frac{5}{2}+\frac{\sqrt{3}}{2}j+\frac{5}{2}+\frac{\sqrt{3}}{2}j)}$$

$$= \frac{(-3+\sqrt{3}j)}{(-5+\sqrt{3}j)(\sqrt{3}j)} = \frac{(-3+\sqrt{3}j)}{-3-5\sqrt{3}j} = \frac{(-3+\sqrt{3}j)(-3+5\sqrt{3}j)}{9+25\cdot 3}$$

$$= \frac{9-15-3\sqrt{3}j-15\sqrt{3}j}{84} = \frac{-6-18\sqrt{3}j}{84} = \frac{-1-3\sqrt{3}j}{14}$$

$$= -\frac{1}{14} - \frac{3\sqrt{3}}{14}j = \frac{1}{\sqrt{7}}\angle -100.89^\circ$$ where $|C_1| = \frac{1}{\sqrt{7}}$, $\angle C_1 = -100.89^\circ$

by 8.71 in textbook.

$$x(t) = C_2 + C_1 e^{p_1 t} + \bar{C}_1 e^{p_2 t}$$

$$= C_2 + C_1 e^{p_1 t} + \bar{C}_1 e^{\bar{p}_1 t}$$

$$= C_2 + 2|C_1| e^{\sigma t} \cos(wt + \angle C_1)$$

$$\Rightarrow x(t) = \frac{1}{7} + \frac{2}{\sqrt{7}} e^{-\frac{5}{2}t} \cos(\frac{\sqrt{3}}{2}t - 100.89^\circ), \quad t \geq 0$$

or $x(t) = \frac{1}{7}u(t) + \frac{2}{\sqrt{7}} e^{-\frac{5}{2}t} \cos(\frac{\sqrt{3}}{2}t - 100.89^\circ)\, u(t)$

(c). $X(s) = \dfrac{2s^2-9s-35}{s^2+4s+2}$ Type: two distinct poles

$s^2+4s+2=0$

$\Rightarrow s^2+4s+4-4+2=0$

$\Rightarrow s_{1,2} = -2 \pm \sqrt{2}$

$$X(s) = \frac{2s^2+8s+4-17s-39}{s^2+4s+2}$$

$$= \frac{2(s^2+4s+2)}{s^2+4s+2} - \frac{17s+39}{s^2+4s+2}$$

$$\Rightarrow X(s) = 2 - \frac{c_1}{s+2-\sqrt{2}} - \frac{c_2}{s+2+\sqrt{2}}$$

$$= 2 - \left(\frac{c_1(s+2+\sqrt{2}) + c_2(s+2-\sqrt{2})}{s^2+4s+2} \right)$$

$$= 2 - \frac{(c_1+c_2)s + (2+\sqrt{2})c_1 + (2-\sqrt{2})c_2}{s^2+4s+2}$$

solve $c_1+c_2=17$ for c_1, c_2

$(2+\sqrt{2})c_1 + (2-\sqrt{2})c_2 = 39$

$$\Rightarrow c_1 = \frac{17}{2} + \frac{5\sqrt{2}}{4}, \quad c_2 = \frac{17}{2} - \frac{5\sqrt{2}}{4}$$

$$\Rightarrow X(s) = 2 - \frac{\frac{17}{2} + \frac{5\sqrt{2}}{4}}{s+2-\sqrt{2}} - \frac{\frac{17}{2} - \frac{5\sqrt{2}}{4}}{s+2+\sqrt{2}}$$

$$\Rightarrow x(t) = 2\delta(t) - \left(\frac{17}{2} + \frac{5\sqrt{2}}{4}\right)e^{(-2+\sqrt{2})t} - \left(\frac{17}{2} - \frac{5\sqrt{2}}{4}\right)e^{(-2-\sqrt{2})t} \quad t \geq 0$$

or $x(t) = 2\delta(t) - \left(\frac{17}{2} + \frac{5\sqrt{2}}{4}\right)e^{(-2+\sqrt{2})t}u(t) - \left(\frac{17}{2} - \frac{5\sqrt{2}}{4}\right)e^{(-2-\sqrt{2})t}u$

approximately, $x(t) = 2\delta(t) - 10.27e^{-0.586t}u(t) - 6.73e^{-3.414t}u(t)$

(f) $X(s) = \dfrac{s+e^{-s}}{s^2+s+1}$ Type : two complex poles

$$X(s) = \frac{s}{s^2+s+1} + \frac{e^{-s}}{s^2+s+1}$$

$$= \frac{s+\frac{1}{2}-\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} + \frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}} e^{-s}$$

$$= \frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} - \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)\frac{(\frac{\sqrt{3}}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} + \left(\frac{2}{\sqrt{3}}\right)\frac{(\frac{\sqrt{3}}{2})}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} e^{-s}$$

$$\Rightarrow x(t) = \left(e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}} e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t\right)u(t) + \frac{2}{\sqrt{3}} e^{-\frac{1}{2}(t-1)} \sin\frac{\sqrt{3}}{2}(t-1)\, u(t-1)$$

$$= \frac{2}{\sqrt{3}} e^{-\frac{1}{2}t}\left(\frac{\sqrt{3}}{2}\cos\frac{\sqrt{3}}{2}t - \frac{1}{2}\sin\frac{\sqrt{3}}{2}t\right)u(t) + \frac{2}{\sqrt{3}}e^{-\frac{1}{2}(t-1)}\sin\frac{\sqrt{3}}{2}(t-1)\, u(t-1)$$

$$\Rightarrow x(t) = \frac{2}{\sqrt{3}} e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t+30^\circ\right)u(t) + \frac{2}{\sqrt{3}}e^{-\frac{1}{2}(t-1)}\sin\frac{\sqrt{3}}{2}(t-1)\, u(t-1)$$

Note $\frac{2}{\sqrt{3}} \approx 1.1547$.

The properties are used.

$y(t-c)\,u(t-c) \leftrightarrow Y(s)\,e^{-cs}$ for $c > 0$.

$(e^{-bt}\cos wt)\,u(t) \leftrightarrow \dfrac{s+b}{(s+b)^2+w^2}$

$(e^{-bt}\sin wt)\,u(t) \leftrightarrow \dfrac{w}{(s+b)^2+w^2}$