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8.5 (a) poles are $s=0, -1$, both are real, and $s \le 0$

so $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{4s}{s^2+s} = 4$

(b) poles are $s=0, -1$, both are real, and $s \le 0$

so $\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,\frac{3s+4}{s^2+s} = \lim_{s\to 0} \frac{3s+4}{s+1} = 4$

(c). Poles are $s=0, 1$, both are real but $s=1>0$

so there is no final value. If you insist on having one, it must be ∞.

Comments: when you apply Final Value Theorem, do check the poles. If poles are real and larger than or equal to 0, it means that the signal is stable. If you get a pole >0, the signal is unstable. $x(t) \xrightarrow[t\to\infty]{} \infty$. If you get a pair of complex poles, $x(t)$ might oscillate and $x(t) \xrightarrow[t\to\infty]{}$ some values (but not stick to only one value).

8.8. (a) $x(t)=e^{-t}u(t) \rightarrow X(s)=\frac{1}{s+1}$ ②

$v(t)=\sin t\, u(t) \rightarrow V(s)=\frac{1}{s^2+1}$

$x(t) * v(t) \longrightarrow X(s)V(s) = \frac{1}{s+1}\,\frac{1}{s^2+1}$

$$\frac{1}{s+1}\,\frac{1}{s^2+1} = \frac{1}{2}\,\frac{1}{s+1} - \frac{1}{2}\,\frac{s}{s^2+1} + \frac{1}{2}\,\frac{1}{s^2+1}$$

$\Rightarrow$ Inverse Laplace $\Rightarrow$ $x(t) * v(t) = \underline{\left(\frac{1}{2}e^{-t} - \frac{1}{2}\cos t + \frac{1}{2}\sin t\right)u(t)}$

or $x(t) * v(t) = \left(\frac{1}{2}e^{-t} - 0.707\cos(t+45^\circ)\right)u(t)$

(c). $x(t) = \sin t\, u(t) \rightarrow X(s) = \frac{1}{s^2+1}$

$v(t) = (t\sin t)u(t) \rightarrow V(s) = \frac{2s}{(s^2+1)^2}$

$x(t) * v(t) \rightarrow X(s)V(s) = \frac{2s}{(s^2+1)(s^2+1)^2} = \frac{2s}{(s^2+1)^3}$

Textbook 384-385 for repeated poles.

Pole $p_1 = j$ and $p_2 = -j$ repeated three times. Thus,

$$X(s)V(s) = \frac{c_1}{(s-j)} + \frac{c_2}{(s-j)^2} + \frac{c_3}{(s-j)^3} + \frac{d_1}{(s+j)} + \frac{d_2}{(s+j)^2} + \frac{d_3}{(s+j)^3} \quad (*)$$

$c_3 = [(s-j)^3 X(s)V(s)]_{s=j}$, $c_{3-i} = \frac{1}{i!}\left[\frac{d^i}{ds^i}[(s-j)^3 X(s)V(s)]\right]\Big|_{s=j}$, $i=1,2.$

$d_3 = [(s+j)^3 X(s)V(s)]\Big|_{s=-j}$, $c_{3-i} = \frac{1}{i!}\left[\frac{d^i}{ds^i}[(s+j)^3 X(s)V(s)]\right]\Big|_{s=-j}$, $i=1,2.$

Or put the right-hand side of (*) over a common denominator and then equating coefficients of the resulting numerator with the numerator of $X(s)$.

Computing gets: $c_1 = 0$, $c_2 = j/8$, $c_3 = -1/4$, $d_1 = 0$, $d_2 = -j/8$, $d_3 = -1/4$

$$\Rightarrow X(s)V(s) = \frac{j/8}{(s-j)^2} - \frac{1/4}{(s-j)^3} - \frac{j/8}{(s+j)^2} - \frac{1/4}{(s+j)^3}$$

Remember $t^N e^{-bt} u(t) \leftrightarrow \frac{N!}{(s+b)^{N+1}}$, $N=1,2,3,\cdots \Rightarrow \frac{t^N e^{-bt}}{N!} u(t) \leftrightarrow \frac{1}{(s+b)^{N+1}}$ 3

$\Rightarrow \frac{1}{(s-j)^2} \leftrightarrow t e^{jt}$, $\frac{1}{(s+j)^2} \leftrightarrow t e^{-jt}$

$\frac{1}{(s-j)^3} \leftrightarrow \frac{t^2 e^{jt}}{2}$, $\frac{1}{(s-j)^3} \leftrightarrow \frac{t^2 e^{-jt}}{2}$

$$\Rightarrow x(t) * v(t) = \frac{j}{8} t e^{jt} - \frac{1}{8} t^2 e^{jt} - \frac{j}{8} t e^{-jt} - \frac{1}{8} t^2 e^{-jt}$$

$$= \frac{j}{8} t (e^{jt} - e^{-jt}) - \frac{1}{8} t^2 (e^{jt} + e^{-jt})$$

$$= \left(-\frac{t}{4} \sin t - \frac{t^2}{4} \cos t\right) u(t), \quad t \geq 0.$$

8.15. (a) Do you remember for the homework 1, I gave the comment $y(t-c)\, u(t-c) \leftrightarrow Y(s) e^{-cs}$ for $c > 0$

Today Let's expand this property by

$$\frac{d^n y(t-c)}{dt^n} u(t-c) \leftrightarrow s^n Y(s) e^{-cs}$$

The differential equation is rewritten as

$$\frac{d^2 y(t)}{dt^2} u(t) + 2 \frac{d y(t-1)}{dt} u(t-1) - y(t) u(t) + 3 y(t-2) u(t-2)$$

$$= \frac{d x(t)}{dt} u(t) + x(t-2) u(t-2)$$

Laplace transform Leads to

$$s^2 Y(s) + 2 s Y(s) e^{-s} - Y(s) + 3 Y(s) e^{-2s}$$

$$= s X(s) + X(s) e^{-2s}$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{s + e^{-2s}}{s^2 + 2 s e^{-s} - 1 + 3 e^{-2s}}$$

(b) Assume $V(s) = \frac{1}{s^2+2se^{-s}-1+3e^{-2s}}$

Remember $\dot{v}(t) \Leftrightarrow sV(s) - v(0^-)$ (Page 369, text)

$\Rightarrow \quad sV(s) \longleftrightarrow \dot{v}(t) + v(0^-)$

Also you know $V(s)e^{-2s} \longleftrightarrow v(t-2)u(t-2)$.

$$\Rightarrow H(s) = \frac{s}{s^2+2se^{-s}-1+3e^{-2s}} + \frac{e^{-2s}}{s^2+2se^{-s}-1+3e^{-2s}}$$

$$\Rightarrow H(s) = \frac{s}{V(s)} + \frac{e^{-2s}}{V(s)}$$

$\Rightarrow$ impulse response $h(t)$

$$h(t) = \dot{v}(t) + v(0^-) + v(t-2)u(t-2)$$

$$h(t) = \frac{dv(t)}{dt} + v(0^-) + v(t-2)u(t-2)$$

To get the exact $v(t)$ is beyond your level. So don't waste your time on trying. In the high level course, we can use Pade approximation to replace e^{-cs} and get our polynomial of $V(s)$.