ee 352 HW04 Solu ①

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11.1 (a) $x(n)$ possesses a Z-transform means

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} \text{ converges}$$

$\because$ $x[n] = \begin{cases} b^n, & n = 0, 1, 2, \cdots N-1 \\ 0 & \text{otherwise} \end{cases}$

$\therefore$ $x[n]$ is a finite impulse response

$$X(z) = \sum_{n=0}^{N-1} x[n] z^{-n} = \sum_{n=0}^{N-1} b^n z^{-n} = \sum_{n=0}^{N-1} (bz^{-1})^n$$

no matter what you choose for bz^{-1}, $X(z)$ is finite and thus $X(z)$ converge.

Or check Text, Page 595,

if $\sum_{n=0}^{\infty} |x[n]| < \infty$, $x[n]$ converges to zero and $X(z)$ exists

$\sum_{n=0}^{N-1} |b^n| < \infty$ $\therefore$ $x[n]$ has a Z transform.

(b). $$X(z) = \sum_{n=0}^{N-1} b^n z^{-n} = \sum_{n=0}^{N-1} \left(\frac{b}{z}\right)^n \quad (1)$$

$$\frac{b}{z} X(z) = \sum_{n=0}^{N-1} \left(\frac{b}{z}\right)^{n+1} = \sum_{n=1}^{N} \left(\frac{b}{z}\right)^n = \sum_{n=0}^{N-1} \left(\frac{b}{z}\right)^n - 1 + \left(\frac{b}{z}\right)^N \quad (2)$$

$$(1) - (2) \Rightarrow \left(1 - \frac{b}{z}\right) X(z) = 1 - \left(\frac{b}{z}\right)^N$$

$$X(z) = \frac{1 - \left(\frac{b}{z}\right)^N}{1 - \frac{b}{z}} = \frac{z^N - z^b}{z^{N-1}(z-b)}$$

11.3 (g)

By def of Z transform

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} u(n) z^{-n} + \sum_{n=0}^{\infty} (-nu(n-1)) z^{-n} + \sum_{n=0}^{\infty} (\tfrac{1}{3})^n u(n-2) z^{-n}$$

$$= X_1(z) + X_2(z) + X_3(z).$$

$$X_1(z) = \sum_{n=0}^{\infty} u(n) z^{-n} = \sum_{n=0}^{\infty} z^{-n} \quad (1)$$

$$zX_1(z) = z\sum_{n=0}^{\infty} z^{-n} = \sum_{n=0}^{\infty} z^{-n+1} = \sum_{n=-1}^{\infty} z^{-n} \quad (2)$$

$$(2)-(1) \Rightarrow (z-1)X_1(z) = z \Rightarrow X_1(z) = \frac{z}{z-1} \quad (3)$$

$$X_2(z) = \sum_{n=0}^{\infty} (-nu(n-1)) z^{-n} = -\sum_{n=1}^{\infty} n z^{-n}$$

$$zX_2(z) = -z\sum_{n=1}^{\infty} n z^{-n} = -\sum_{n=1}^{\infty} n z^{-n+1} = -\sum_{n=0}^{\infty} (n+1) z^{-n}$$

$$zX_2(z) - X_2(z) = -\sum_{n=0}^{\infty} z^{-n} = -\frac{z}{z-1} \text{ from (3)}$$

$$X_2(z) = -\frac{z}{(z-1)^2}$$

$$X_3(z) = \sum_{n=0}^{\infty} (\tfrac{1}{3})^n u(n-2) z^{-n} = \sum_{n=2}^{\infty} (3z)^{-n} = -(3z)^{-0} - (3z)^{-1} + \sum_{n=0}^{\infty} (3z)^{-n}$$

$$= -1 - (3z)^{-1} - \frac{3z}{3z-1} \text{ from (3)} = \frac{3z-1+1-(3z)^{-1}-3z}{3z-1}$$

$$= -\frac{(1+(3z)^{-1})(3z-1)-3z}{3z-1} =$$

$$= -\frac{-(3z)^{-1}}{3z-1} = \frac{1}{9z^2-3z}$$

Or check text, page 572, table 11.3.

$$x(n) = u(n) - nu(n-1) + (\tfrac{1}{3})^n u(n-2)$$

$$= u(n) - u(n-1) - (n-1)u(n-1) + \tfrac{1}{9}(\tfrac{1}{3})^{n-2} u(n-2)$$

$$= \frac{z}{z-1} - \frac{z}{z-1} z^{-1} - \frac{z}{(z-1)^2} z^{-1} + \frac{1}{9} \frac{z}{z-\frac{1}{3}} z^{-2}$$

$$= \frac{z}{z-1} - \frac{1}{z-1} - \frac{1}{(z-1)^2} + \frac{1}{9z^2-3z}$$

The properties are used :

$x[n-q]\,u(n-q) \leftrightarrow z^{-q}X(z)$ $\quad q$: a positive integer

$(n-1)\,u(n-1) \leftrightarrow \frac{z}{(z-1)^2}z^{-1}$

$a^n u(n) \leftrightarrow \frac{z}{z-a}$ $\quad a$: a complex number

$a^{n-q}u(n-q) \leftrightarrow \frac{z}{z-a}z^{-q}$: q : a positive integer.

$a^{n-1}u(n-1) \leftrightarrow \frac{z}{z-a}z^{-1}$

(j) By def,

$$X(z) = \sum_{n=0}^{\infty} x(n)\,z^{-n} = \sum_{n=0}^{\infty} (0.25)^{-n}u(n-2)\,z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{4}\right)^{-n}u(n-2)\,z^{-n} = \sum_{n=2}^{\infty} \left(\tfrac{1}{4}\right)^{-n}z^{-n}$$

$$= \sum_{n=2}^{\infty} \left(\tfrac{z}{4}\right)^{-n} = \sum_{n=0}^{\infty} \left(\tfrac{z}{4}\right)^{-n} - \left(\tfrac{z}{4}\right)^{0} - \left(\tfrac{z}{4}\right)^{-1}$$

$$= \frac{z/4}{z/4-1} - 1 - \frac{4}{z} = \frac{\frac{z}{4} - \left(\frac{z}{4}-1\right)\left(1+\frac{4}{z}\right)}{\frac{z}{4}-1}$$

$$= \frac{\frac{z}{4} - \frac{z}{4} - 1 + 1 + \frac{4}{z}}{\frac{z}{4}-1} = \frac{\frac{4}{z}}{\frac{z}{4}-1} = \frac{16}{z^2-4z}$$

Or check the table,

$$x(n) = \left(\tfrac{1}{4}\right)^{-n}u(n-2) = \left(\tfrac{1}{4}\right)^{-2}\left(\tfrac{1}{4}\right)^{-n+2}u(n-2) = 16\,(4)^{(n-2)}u(n-2)$$

$$\Rightarrow X(z) = 16 \cdot \frac{z}{z-4}z^{-2} = \frac{16}{z^2-4z}$$

use the property $a^{n-q}u(n-q) \leftrightarrow \frac{z}{z-a}z^{-q}$

11.8 $X(z) = \frac{z+1}{z(z-1)} = -\frac{1}{z} + \frac{2}{z-1}$

Property used

$\frac{1}{z^q} \Leftrightarrow \delta(n-q),\ q=1,2\cdots$

$\frac{z}{z-1} \Leftrightarrow u(n)$

$z^{-1}X(z) \Leftrightarrow x(n-1)u(n-1)$

$X(z) = -\frac{1}{z} + z^{-1}\frac{2z}{z-1}$

$\Rightarrow \quad x(n) = -\delta(n-1) + 2u(n-1)$

$x(0) = -\delta(0-1) + 2u(0-1) = 0$

$x(1) = -\delta(1-1) + 2u(1-1) = -1+2 = 1$

$x(10000) = -\delta(10000-1) + 2u(10000-1) = 2$

11.9 (e) $X(z) = \frac{z^2-1}{z^2+1} = \frac{z^2}{z^2+1} - \frac{1}{z^2+1} = \frac{z^2}{z^2+1} - \frac{z}{z^2+1}z^{-1}$

$\Rightarrow x(n) = (\cos\frac{\pi}{2}n)u(n) - \sin\frac{\pi}{2}(n-1)\,u(n-1)$

$x(n) = (\cos\frac{\pi}{2}n)u(n) + (\cos\frac{\pi}{2}n)u(n-1)$

$x(n) = \delta(n) + (2\cos\frac{\pi n}{2})u(n-1)$

Properties used: $(\cos\Omega n)u(n) \leftrightarrow \frac{z^2-(\cos\Omega)z}{z^2-(2\cos\Omega)z+1}$

$(\sin\Omega n)u(n) \leftrightarrow \frac{(\sin\Omega)z}{z^2-(2\cos\Omega)z+1}$

$z^{-1}X(z) \Leftrightarrow x(n-1)u(n-1)$

11.14. By definition of $H(z)$

$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} = \sum_{n=-\infty}^{0} 0\cdot z^{-n} + \sum_{n=1}^{3}\frac{1}{n}z^{-n} + \sum_{n=4}^{5}(n-2)z^{-n}$

$= 0 + z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{3}z^{-3} + 2z^{-4} + 3z^{-5}$

$= \frac{z^4 + \frac{1}{2}z^3 + \frac{1}{3}z^2 + 2z^1 + 3}{z^5}$

11.15 $x(n) = (-1)^n u(n)$ (5)

$$X(z) = \sum_{n=0}^{\infty} (-1)^n u(n) z^{-n} = \sum_{n=0}^{\infty} u(n) (-z)^{-n}$$

$$X(z) = \frac{-z}{-z-1} = \frac{z}{z+1}$$

$$y(n) = \begin{cases} 0 & n<0 \\ n+1 & n=0,1,2,3 \\ 0 & n \geq 4 \end{cases}$$

$$\Rightarrow Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} = 1z^{-0} + 2z^{-1} + 3z^{-2} + 4z^{-3}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}+3z^{-2}+4z^{-3}}{z/(z+1)}$$

$$H(z) = \frac{z^4+3z^3+5z^2+7z+4}{z^4}$$