# ee352 HW06 Solu

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13.7. From the block diagram,

$$Y(s) = \frac{5}{s+1}\left(\frac{2}{s+3} - \frac{1}{s+2}\right)V(s)$$

$$= \frac{5}{s+1}\left(\frac{2(s+2)-s-3}{(s+3)(s+2)}\right)V(s)$$

$$= \frac{5}{s+1}\,\frac{s+1}{(s+2)(s+3)}V(s)$$

$$\Rightarrow Y(s) = \frac{5}{(s+2)(s+3)}V(s)$$

There are millions of ways to construct the state model:

① method 1. For this special transform, we can do it by differential equation.

$$Y(s)(s^2+5s+6) = 5V(s)$$

$$\Rightarrow \ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 5v(t)$$

select $q_1(t) = y(t)$,
$q_2(t) = \dot{y}(t)$

$$\Rightarrow \dot{q}_1(t) = \dot{y}(t) = q_2(t)$$

$$\dot{q}_2(t) = \ddot{y}(t) = -5\dot{y}(t) - 6y(t) + 5v(t)$$

$$= -5q_2(t) - 6q_1(t) + 5v(t)$$

$$\begin{pmatrix}\dot{q}_1(t)\\ \dot{q}_2(t)\end{pmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}}_{=A}\begin{bmatrix}q_1(t)\\ q_2(t)\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 5\end{bmatrix}}_{=B}v(t)$$

②

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \underbrace{0}_{D} \cdot V(t)$$

② method 2. Using paralell circuit.

$$Y(s) = \frac{5}{(s+2)(s+3)} V(s) = \left( \frac{a}{s+2} + \frac{b}{s+3} \right) V(s)$$

$$= \left( \frac{5}{s+2} - \frac{5}{s+3} \right) V(s)$$

$$= \frac{5}{s+2} V(s) - \frac{5}{s+3} V(s)$$

$$= 5 X_1(s) - 5 X_2(s)$$

$$X_1(s) = \frac{1}{s+2} V(s), \quad X_2(s) = \frac{1}{s+3} V(s)$$

$$\Rightarrow \dot{x}_1(t) = -2x_1(t) + V(t),$$

$$\dot{x}_2(t) = -3x_2(t) + V(t)$$

$$y(t) = 5x_1(t) - 5x_2(t)$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{B} V(t)$$

$$y(t) = \underbrace{\begin{bmatrix} 5 & -5 \end{bmatrix}}_{C} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0\, V(t)$$

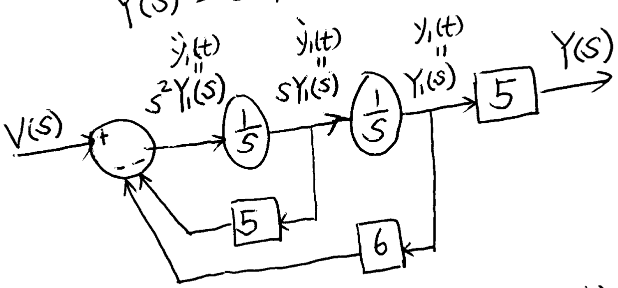
③ methed 3: "Control" canonical form.

$$\frac{Y(s)}{V(s)} = \frac{5}{(s+2)(s+3)} = \frac{Y(s)}{Y_1(s)}\,\frac{Y_1(s)}{V(s)}$$

$$\Rightarrow \quad \frac{Y(s)}{Y_1(s)} = 5, \qquad \frac{Y_1(s)}{V(s)} = \frac{1}{(s+2)(s+3)}$$

$$\Rightarrow \qquad Y_1(s)\,s^2 = -5Y_1(s)\,s - 6Y_1(s) + V(s)$$

$$Y(s) = 5\,Y_1(s)$$




$$y_1(t) = q_1(t), \quad \dot{y}_1(t) = q_2(t) \quad \Rightarrow \quad \begin{aligned} \dot{q}_1(t) &= \dot{y}_1(t) = q_2(t) \\ \dot{q}_2(t) &= -5q_2(t) - 6q_1(t) + v(t) \end{aligned}$$

$$\Rightarrow \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}}_{=A} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{=B} V(t)$$

$$y(t) = 5y_1(t) = \underbrace{5}_{=C}\, q_1(t) + \underbrace{0}_{=D}\, V(t)$$

This method is useful when the orders of the numerator and denominator are the same. e.g. $\frac{Y(s)}{V(s)} = \frac{s^2+s+1}{3s^2+4s+5}$.

I don't want to try the other methods and stop here.

13.16. (b).

$\dot{x}(t) = A x(t) + B V(t) \Rightarrow sX(s) = A X(s) + BV(s)$ ①

$y(t) = C x(t) \Rightarrow Y(s) = C X(s)$ ②

①$\Rightarrow (IS - A) X(s) = BV(s)$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow X(s) = (IS - A)^{-1} B V(s)$

$\Rightarrow Y(s) = C(IS - A)^{-1} B V(s)$

Know, $C$, $B$, want to know $(IS - A)^{-1}$

$IS - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} s - \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$

$IS - A = \begin{bmatrix} s & -2 \\ 0 & s+1 \end{bmatrix}$

How do we calculate the inverse of a matrix?

one way =

$\left[\begin{array}{cc|cc} s & -2 & 1 & 0 \\ 0 & s+1 & 0 & 1 \end{array}\right] \Leftrightarrow \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{s} & \frac{2}{s(s+1)} \\ 0 & 1 & 0 & \frac{1}{s+1} \end{array}\right]$

By elementary operation of the rows of a matrix

Then $\begin{bmatrix} \frac{1}{s} & \frac{2}{s(s+1)} \\ 1 & \frac{1}{s+1} \end{bmatrix} = (IS - A)^{-1}$

Or $(IS - A)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow a = \frac{1}{s},\ b = \frac{2}{s(s+1)},\ c = 0,\ d = \frac{1}{s+1}$

$\Updownarrow$ By 4 variables, 4 equations.

Or simply remember the following formula for a 2X2 matrix inverse

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} / (ad - bc)$

Now know $C$, $(IS-A)^{-1}$, $B$,

$$H(s)=C(IS-A)^{-1}B = \begin{bmatrix} 1 & 3 \end{bmatrix}\begin{bmatrix} \frac{1}{s} & \frac{2}{s(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$H(s)=\begin{bmatrix} \frac{4s+3}{s(s+1)} , & \frac{-(3s+2)}{s(s+1)} \end{bmatrix}$$

Take it easy, $H(s)$ is a row vector because this system is single input multiple output.

If you study advanced linear control, you will encounter those systems.